TIME VALUE OF MONEY

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THE CONCEPT OF TIME VALUE OF MONEY

- An individual's preference for possession of a given amount of cash now, than the same amount in future is called 'time preference for money'.
- Three reasons that explain the time value of money:
  - Investment opportunities
  - Preference for present consumption
  - Risk or uncertainty
- Two methods for accounting for time value of money: **Compounding and Discounting.**

EFFECTIVE AND NOMINAL RATE OF INTEREST

- **NOMINAL INTEREST RATE:** Rate specified on an annual basis on a loan agreement or security
- **EFFECTIVE INTEREST RATE**: The actual rate of interest paid when compounding is done more than once a year.
- Effective interest rate > Nominal Interest rate
- Relationship between effective and nominal interest rate

$$r = \left(1 + \frac{k}{m}\right)^m - 1$$

- where, r is the effective rate of interest

  k is the nominal rate of interest

  m is the frequency of compounding per year.

- **Example:** On a particular saving scheme, a bank is offering an interest rate of 9% compounded quarterly. What is the effective rate of interest being offered by the bank?
- **Solution:**

The effective rate of interest can be computed as:

$$r = \left(1 + \frac{k}{m}\right)^m - 1$$

$$= \left(1 + \frac{0.09}{4}\right)^4 - 1 = 9.31\% \text{ p.a. compounded quarterly.}$$

METHOD OF COMPOUNDING

- The process of accumulating interest in an investment over time to earn more interest is known as compounding.

- Compounding helps us in finding the future value of a payment (or receipt) or a series of payments (or receipts).

FUTURE VALUE INVESTMENT FACTOR

- The future value of a single cash flow at a particular time in future can be computed as

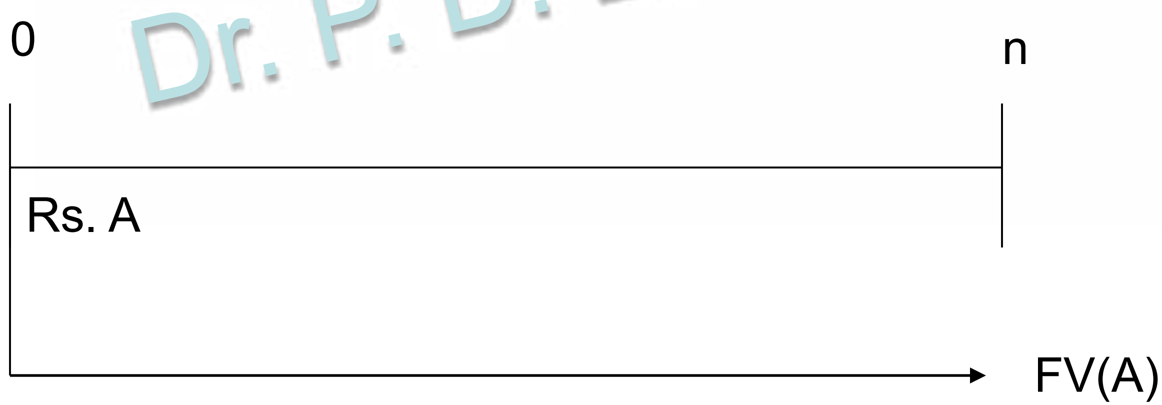
$$FV_n = A\,(1+k)^n$$

where, k : Nominal rate of interest p.a.

FV_n : Future value of the investment at the end of n years

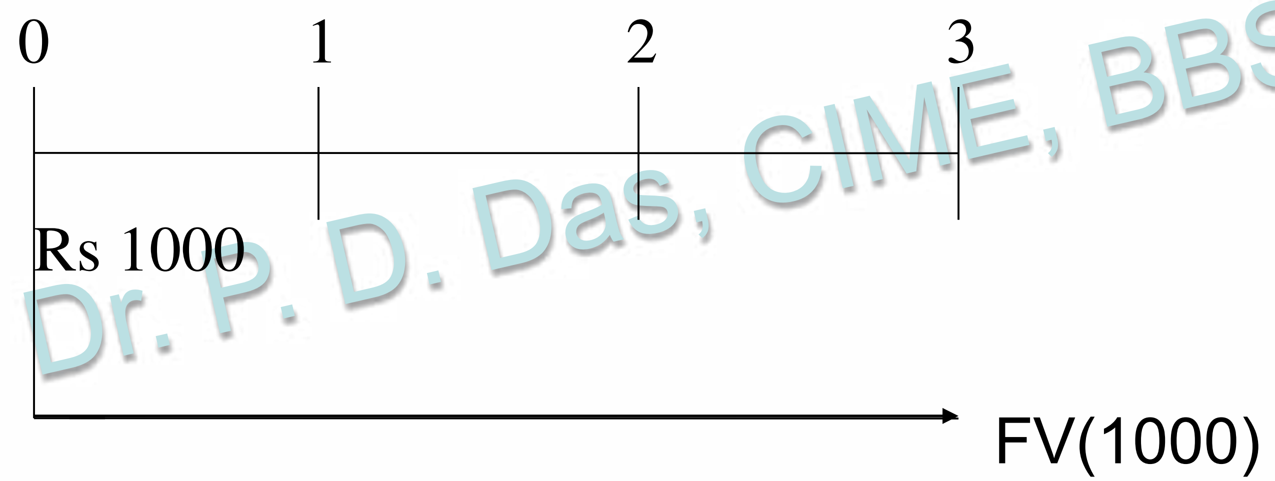
A : Initial cash flow ; n is the life of investment

$(1+k)^n$: Future Value investment factor



- **Example:** Let us find the value of Rs 1,000 (which we have invested now), at the end of 3 years given that the rate of interest earned by it is 4%.
- **Solution:** Future value = Present value $(1+k)^n$

Future value = $1000\,(1+0.04)^3$ = Rs 1,124.86.



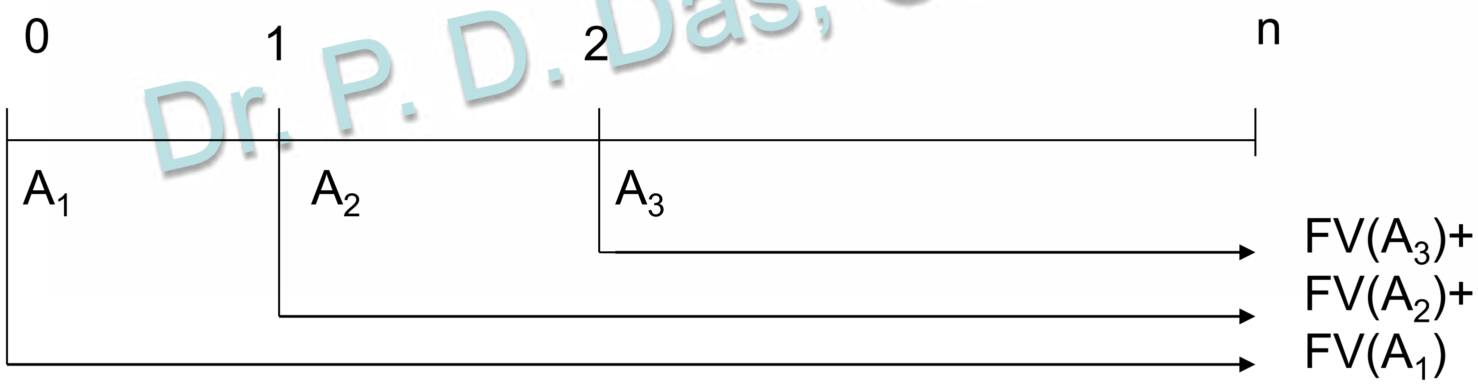
FUTURE VALUE OF MULTIPLE FLOWS

- The future value of multiple flows can be computed as

$$FV_n = A_1 (1+k)^n + A_2 (1+k)^{n-1} + A_3(1+k)^{n-2}$$

where A_1 , A_2 and A_3 are the investments at the beginning of the year 1, 2 …..and 3 respectively.

FV_n : Future value of the investment at the end of n years

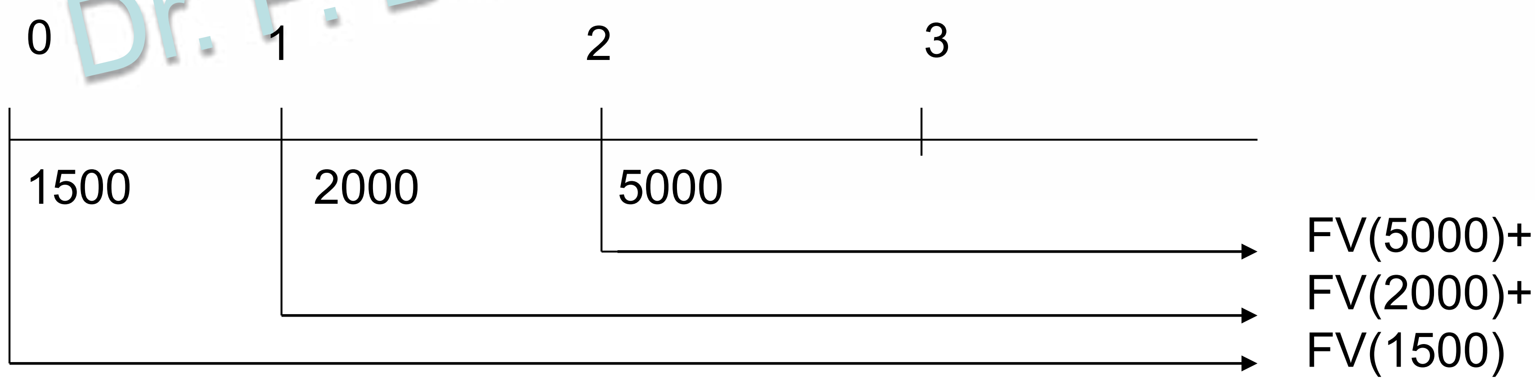


- **Example:** Ram invests Rs 1500 at the beginning of the first year (or in other words at the end of 0^{th} year); Rs. 2,000 at the beginning of the second year and Rs 5,000 at the beginning of third year at a rate of interest 5% per annum. What will be the accumulated value of all these cash outflows at the end of the third year?
- **Solution:**

The accumulated value which Ram will get at the end of three years will be:

$= 1{,}500\ (1+.05)^3 + 2{,}000\ (1+0.05)^2 + 5{,}000\ (1+0.05)^1$

$= 1{,}500\ (1.158) + 2{,}000\ (1.1025) + 5{,}000\ (1.05) = 1737 + 2205 + 5250 = \text{Rs } 9{,}192.$

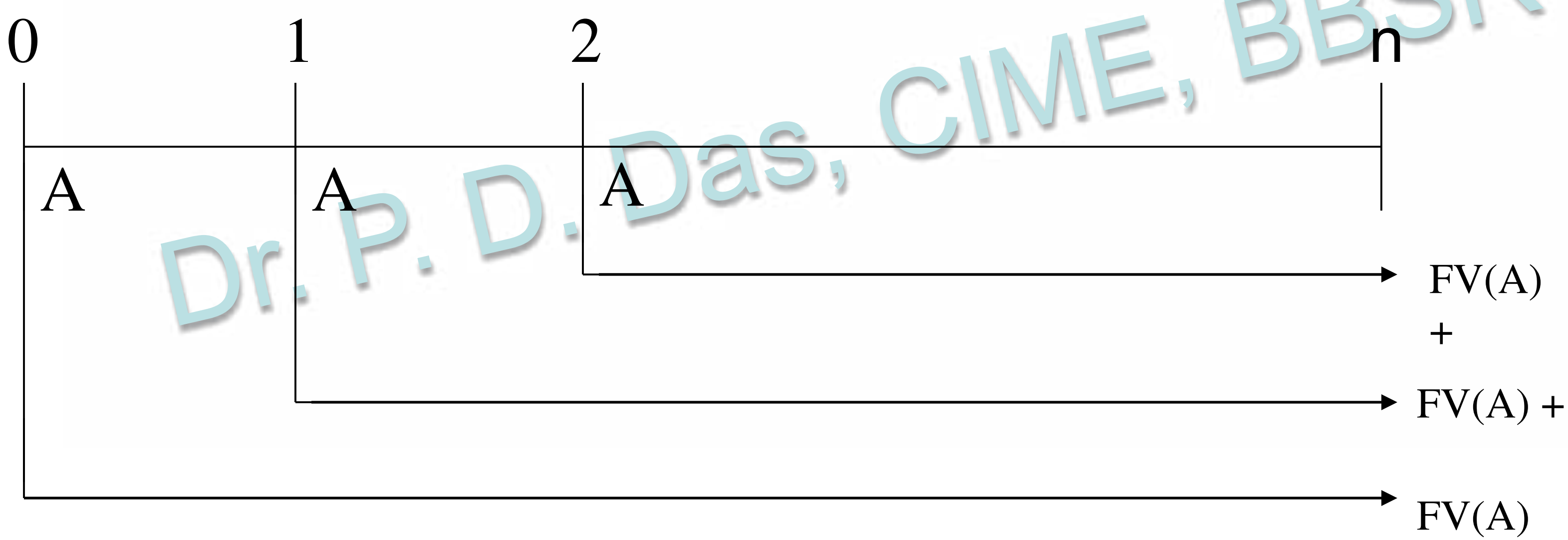


FUTURE VALUE OF AN ANNUITY

- **Annuity** is a pattern of cash flows that are equal in each year.
- **Future value of an annuity**:

$$FVA_n = A\,(1+k)^n + A\,(1+k)^{n-1} + \ldots\ldots + A = A\,\frac{(1+k)^n - 1}{k}$$

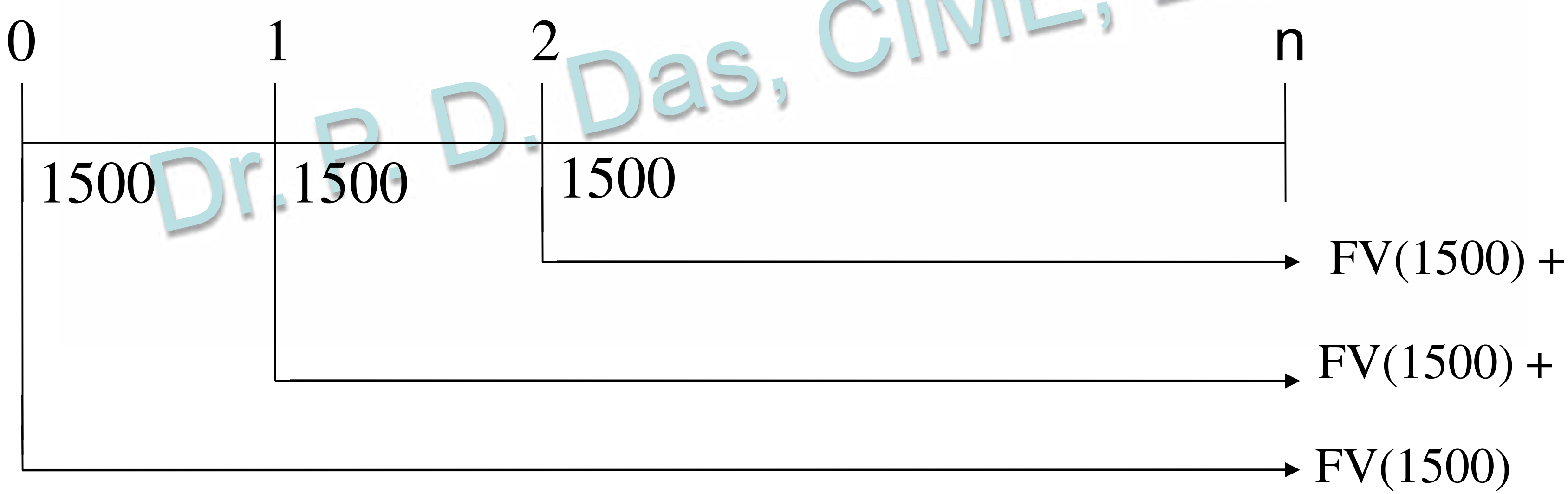
where FVIFA = $[(1+k)^n - 1]/k$



- **Example:** What will be the accumulated value which Vishal will receive at the end of the third year if he invests Rs. 1500 at the beginning of first, second and third year?

- **Solution:** The accumulated value which Ram will get at the end of three years = 1500 FVIFA (5%, 3)

$= 1500\ ((1+.05)^3-1)/.05 = 1500 \times 3.1525 = \text{Rs } 4{,}728.75.$



SINKING FUND FACTOR

- It helps to compute the amount that has to be invested at the end of every year for a period of "n" years at k% rate of interest, in order to accumulate a given amount at the end of the period.
- It is the inverse of the FVIFA.
- Sinking fund factor = $\dfrac{k}{(1+k)^{n}-1}$

- **Example:** A Ltd. has to repay Rs. 55,000 worth debentures at the end of 5 years from now. How much should the firm deposit each year at an interest rate of 5% so that it grows to Rs 55, 000 at the end of the fifth year?

- **Solution:** With the help of sinking fund factor, the amount to be deposited each year can be computed as:

$$A = 55000 \times \frac{0.05}{(1+0.05)^5 - 1} = \text{Rs } 9{,}954.$$

METHOD OF DISCOUNTING

- The process of determining present value of a future payment (or receipts) or a series of future payments is called discounting. The compound interest rate used for discounting the cash flows is also called the discount rate.

PRESENT VALUE OF A SINGLE FLOW

- The present value of an amount expected at some time in future is calculated as:

$$PV = \frac{A}{(1+k)^n} \quad ; \text{where PVIF} = \frac{1}{(1+k)^n}$$

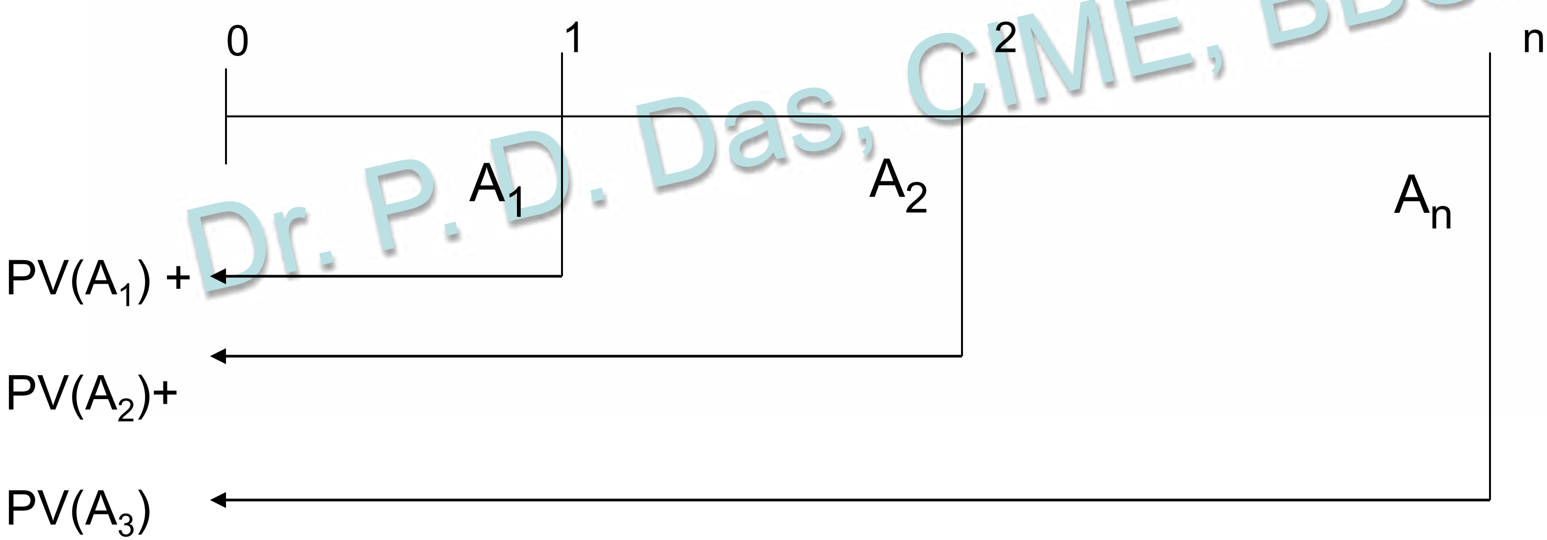
- **Example:** Suppose a particular investment opportunity provides us Rs 2000 at the end of three years. What is the present value of this cash inflow that we will get at the end of three years with the interest rate being 5% ?
- **Solution:** Present value = Future value x $\dfrac{1}{(1+k)^n}$

$= 2000 \quad x \ \dfrac{1}{(1+0.05)^3} = \text{Rs } 1{,}727.68.$

PRESENT VALUE OF MULTIPLE FLOWS

- If A_1, A_2, A_n are the cash flows occurring at the end of the time period 1,2 and n respectively then their present value can be computed as:

$$PV = A_1/(1+k) + A_2/(1+k)^2 + \ldots\ldots\ldots + A_n/(1+k)^n$$

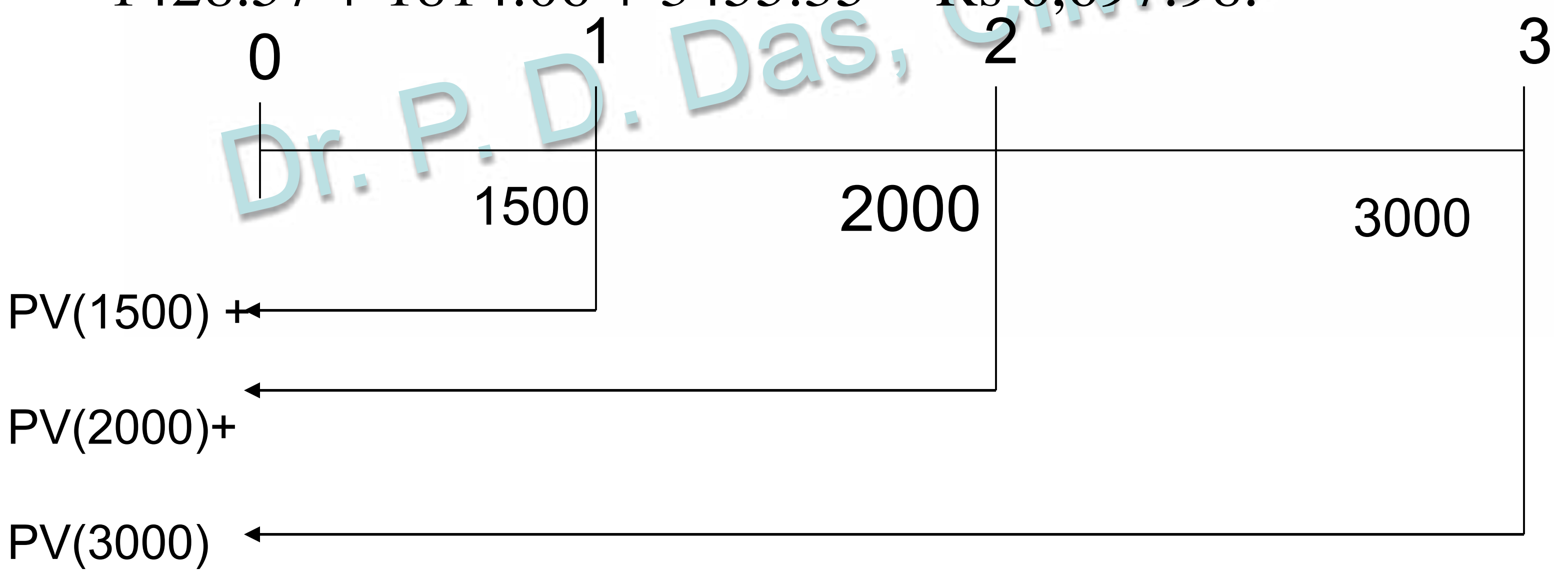


- **Example**: A person invested certain amount of money in a project. The project generates an inflow of Rs 1500 at the end of first year, Rs 2,000 at the end of second year and Rs 4,000 at the end of third year. What is the present value of these future cash inflows given that the rate of interest is 5%?

- **Solution:** Present value = 1500 PVIF (5%,1) + 2000 PVIF(5%,2) + 4000 PVIF(5%,3)

$= 1500 \text{ x } 1/(1+0.05)^1 + 2000 \text{ x } 1/(1+0.05)^2 + 4000 \text{ x } 1/(1+0.05)^3$

$= 1428.57 + 1814.06 + 3455.35 = \text{Rs } 6{,}697.98.$

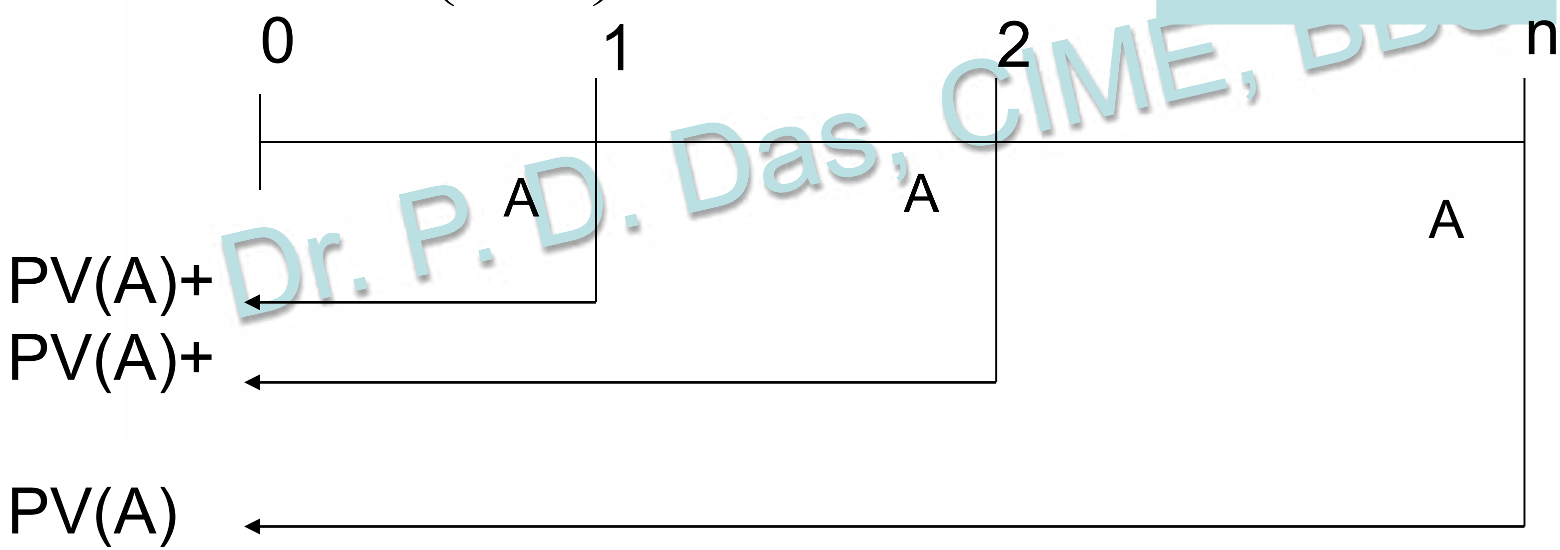


PRESENT VALUE OF AN ANNUITY

- The present value of an annuity can be computed as:

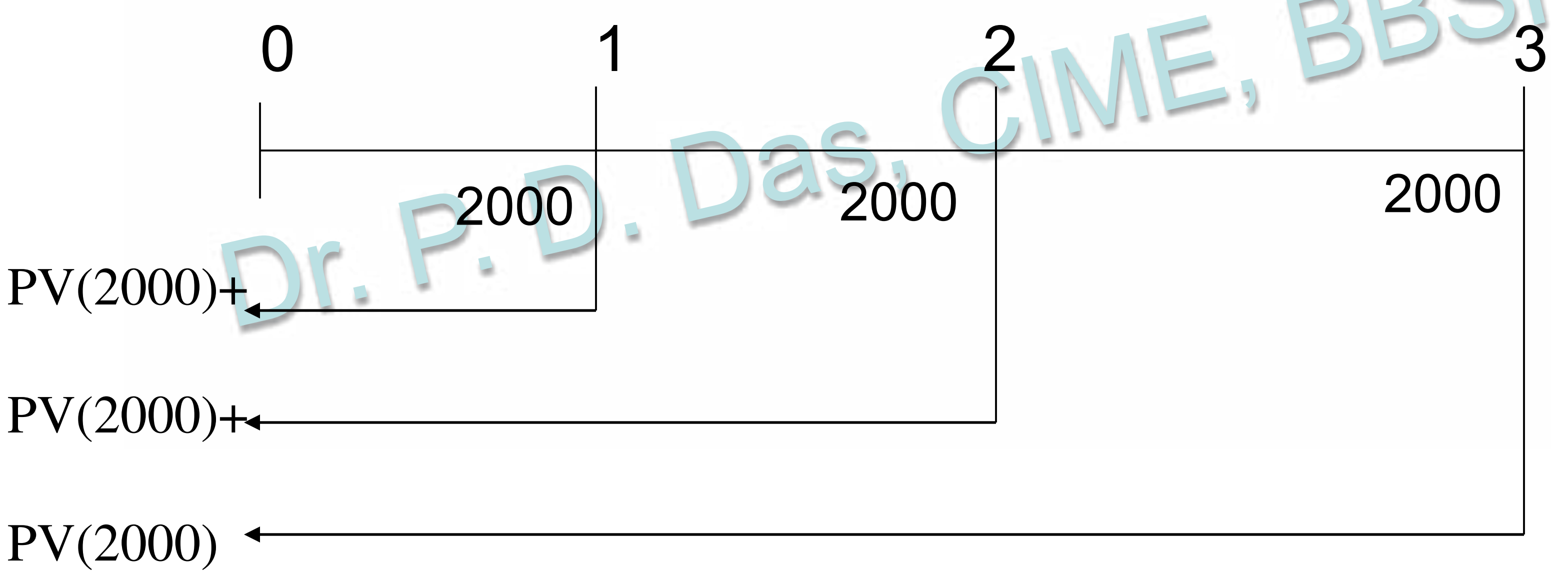
$PV = A/(1+k) + A/(1+k)^2 + \ldots\ldots + A/(1+k)^n$

$PV = A \times \dfrac{(1+k)^n - 1}{k(1+k)^n}$;where PVIFA= $\dfrac{(1+k)^n - 1}{k(1+k)^n}$



- **Example:** A person invested certain amount of money in a project. The project generates an inflow of Rs 2000 at the end of first, second and third year. What is the present value of this annuity of Rs 2000?
- **Solution:** Present Value = 2000 x $\dfrac{(1+0.05)^3 - 1}{0.05(1+0.05)^3}$

= 2000 x 2.7232 = Rs 5,446.40.



CAPITAL RECOVERY FACTOR

- Capital Recovery Factor helps in computing:
  (1) Loan installment to liquidate a loan
  (2) Amount that can be withdrawn periodically when a particular amount is invested now.
- The Capital Recovery Factor is the inverse of PVIFA
- Capital Recovery Factor = $\dfrac{k(1+k)^n}{(1+k)^n - 1}$

- **Example:** Ananya borrowed a loan of Rs 14,000 at a rate of 9% for a period of three years. Prepare a loan amortization schedule using the given data.

- **Solution:**

The annual installment for a loan of Rs. 14,000 at a rate of 9% can be computed using the capital recovery factor.

Annual installment = 14,000 x $\dfrac{.09(1.09)^3}{(1.09)^3 - 1}$

= 14,000 x 0.3951 = Rs. 5531

LOAN AMORTIZATION SCHEDULE

End of Year	Annual installment	Interest Payment	Principal repayment	Outstanding Balance
1	5531	1260	4271	9729
2	5531	876	4655	5074
3	5531	457	5074	0

PRESENT VALUE OF A PERPETUITY

- Perpetuity: An annuity with an infinite duration.
- Present value of a perpetuity=

$$P_{\infty} = A \times \frac{1}{k}$$

where A is the constant annual payment.

•Perpetuity

Immediate Perpetuity

(Payment made at the end of each period)

Perpetuity Due

(Payment made at the beginning of each period)

- **Example:** Ashok has made an investment in an irredeemable debenture that will pay him an annuity of Rs 650 perpetually. Compute the present value of this irredeemable debenture if the rate of interest is 8%.

- **Solution:** The present value of the irredeemable debenture can be computed as:

$$P_{\infty} = A \times \frac{1}{k} = 650 \times \frac{1}{0.08} = \text{Rs. } 8{,}125.$$

END OF SESSION

for any query,
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